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NUMERICAL EXPERIMENTS ON HYSTERETIC PHENOMENA OF SHOCK WAVES IN SUPERSONIC NOZZLE

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ABSTRACT

Recently, hysteretic phenomena in fluid flow systems draw attention of many researchers for their industrial and engineering applications. From some previous researches, the hysteretic phenomenon for the reflection type of shock wave in two-dimensional expanded supersonic jet is occurred under the quasi-steady flow and the transitional pressure ratio between the regular reflection and Mach reflection in the jet is affected by this phenomenon. However, so far, there are very few researches for the hysteretic phenomenon of a supersonic internal flow with shock waves and the phenomenon has not been investigated satisfactorily. The present study pertains with numerical investigation of the hysteretic phenomena for the shock wave in a supersonic nozzle. The location and strength of shock waves in the supersonic nozzle were measured to study the detail about hysteretic phenomena. Moreover, experimental results were presented to validate the present numerical works.

Keywords: Compressible Flow, Hysteresis, Laval Nozzle, Shock Waves, Supersonic.

1. INTRODUCTION

Every process in this universe is irreversible. If a system has any reversed process, the system and surrounding would not return to their original conditions. Hysteretic phenomenon is itself an example of irreversible process. Hysteretic phenomena commonly occur in magnetic and ferromagnetic materials, as well as in the elastic, electric, and magnetic behavior of materials, in which a lag occurs between the application and the removal of a force or field and its subsequent effect. The term ‘hysteresis’ is sometimes also used in other fields, such as economics or biology; where it describes a memory, or lagging effect, in which the order of previous events can influence the order of subsequent events. Hysteresis was initially seen as problematic, but is now thought to be of great importance in technology.

In recent years, hysteretic phenomena in fluid flow systems draw attention of many researchers for their great variety of industrial and engineering applications. During the formation of flow in a certain condition, the flow will experience a transient state until the state and to return to the original state. In general, even a flow changes under the quasi-steady, the flow characteristics is considered to accompany a hysteresis phenomenon (hysteresis loop). In case of a process of rapid change in the flow, this is distinguished from the phenomena for the delay in response time. Hysteresis phenomenon is well known for external flow such as reflection of shock waves in the jet. From some previous researches, it was clarified that when the high-pressure gas is exhausted to atmosphere from the nozzle exit, the expanded supersonic jet with the Mach disk is formed at a specific condition. The jet structure has been known as a fundamental phenomenon of the supersonic fluid mechanics. This type of jet is very important for some industrial devices [1, 2]. Teshima [3] has suggested the possibility of occurrence of the hysteresis in the supersonic jet formed by the rectangular orifice. Chpoun and Ben-Dor [4] numerically confirmed the hysteresis phenomenon in the regular to the Mach reflection transition in steady flows. After that, many researchers have proven that the hysteresis phenomenon exists on the interference form of the shock wave in the flow. On the interference form of oblique shock wave of the two-dimensional flow, the hysteresis phenomenon on the mutual transition of the regular reflection and Mach reflection is mainly reported from the relationship between Mach number and incidence shock wave angle of the flow [4-8]. Gribben et al. [9] have shown for the circular jet that the hysteresis exits on the relation between the location of Mach disk and pressure ratio, and this phenomenon is influenced by the time history effect of plume development. In the recent studies, it is reported that in two-dimensional supersonic jet, the hysteresis phenomenon for the reflection type of shock wave is occurred under the quasi-steady flow of jet and for instance, the transitional pressure ratio between the regular reflection and Mach reflection is affected by this phenomenon [10-12]. Irie et al. [13] observed such hysteresis phenomenon of the under-expanded dry air jet is produced during the transient processes of jet pressure ratio, in which the jet flow obtained in the startup transient is different from that in the shutdown transient. Most recently, the effect of non-equilibrium condensation on hysteresis phenomenon of under-expanded moist jets have investigated [14] and found that the under-expanded moist air jet leads to less hysteresis of the jet, compared with the dry air jets. However, so far, there are very few researches for the hysteresis phenomenon of a supersonic internal flow for shock waves and the phenomenon has not been investigated satisfactorily. From some previous experimental and numerical studies, it realized that in the supersonic part of a Laval nozzle the shock wave is exist, and in some cases normal shock also formed which results a sudden rise in temperature across the shock. The non-homogeneous temperature increase across the shock, caused by the boundary layer thickening ahead of the shock and the resulting pre-compression prevents the quasi 1-D evolution of the flow downstream [15]. Additionally, due to multiple boundary layer interactions the single shock disintegrates into a so called pseudo-shock system; i.e., into a sequence of periodic weak compression and expansion regions [16]. On increasing the nozzle pressure ratio i.e. ratio of pressures between the inlet and exit pressure of the nozzle, the shock position is moved towards the nozzle exit, and vice versa of occurrence of this phenomenon is also true. Therefore, it requires performing in-depth investigation on the hysteresis phenomena for the shock wave in a supersonic nozzle. The purpose of this study is to clarify the hysteresis phenomena for the shock wave in a supersonic nozzle numerically. Hysteresis phenomena for the location and strength of shock wave in a supersonic nozzle were investigated numerically. Moreover, some experimental works were conducted to make confirm the existence of hysteresis phenomenon.

2. EXPERIMENTAL WORKS

2.1 Experimental setup

Figure 1 and 2 shows the schematic diagram of the experimental apparatus and detailed nozzle configuration used in the present study, respectively. The apparatus is consisted of compressor, air drier, air reservoir, electronic control valve, plenum chamber and nozzle. Plenum chamber is placed upstream of the convergent nozzle. The compressed air was used as a working gas. The test section is placed downstream of the nozzle throat and optical glass windows are installed on both the side walls of the test section for flow visualization. In the present study, the nozzle was designed with the method of characteristics. The design Mach number of supersonic nozzle is M_e=2.0 with the throat and exit diameters are D_t=6.0 and D_e=10.0 mm, respectively.

2.2 Experimental method

In the present experiments, pressure ratio $(\phi = p_0/p_b)$ was continuously changed with time using the electronic control valve. The symbol, p_0 and p_b represent the stagnation pressure of the plenum chamber and back pressure (atmospheric pressure), respectively. The shock wave is observed in the divergent part of the nozzle in the range from ϕ =1.42 to 2.02. The rate of the change of pressure ratio with time $\Delta\dot{\phi}$ was changed from 0.112 (1/s) to 0.338 (1/s). Compressed dry air is discharged from nozzle exit through the plenum chamber. The flow field was investigated by a schlieren optical method. Visualization and measurement of pressure ratio were conducted simultaneously. The location of the first shock wave L was obtained from schlieren pictures. In the present experimental conditions, it is found that the time delay exists for response of change of flow to change of pressure ratio at $\Delta\dot{\phi} > 0.318$ (1/s).

3. NUMERICAL ANALYSIS

3.1 Numerical method

Computational Fluid Dynamic (CFD) investigations were performed to investigate the hysteretic phenomena of shock waves in the supersonic nozzle. The nozzle configuration is same as illustrated in Fig. 2. The flow under study was treated as compressible, viscous, unsteady and turbulent. The governing equations are given by the conservation forms of mass, momentum and energy. The mass averaged, time-dependent Navier-Stokes equations are employed in the present computation. The resulting equations are expressed in an integral form:

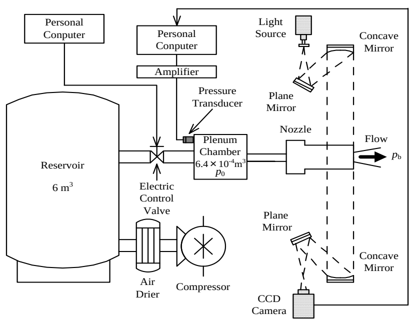


Fig 1. Experimental apparatus

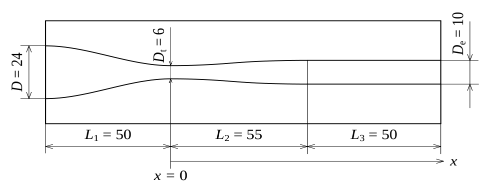


Fig 2. Nozzle geometry (Unit: mm)

$$\Gamma \partial/\partial t\left(\int_{v} \boldsymbol{Q}dV\right)+\oint[\boldsymbol{F}-\boldsymbol{G}]dA=0 \tag{1}$$

where $\boldsymbol{F}$ and $\boldsymbol{G}$ are the inviscid and viscous flux vectors in standard conservation form and $\boldsymbol{Q}$ is the dependent vector of primary variables.

$$\begin{aligned} \boldsymbol{F} &= \left[\rho v, \rho v v_x + p\hat{i}, \rho v v_y + p\hat{j}, \rho v v_z + p\hat{k}, \rho v H\right]^T \\ \boldsymbol{G} &= \left[0, \tau_{xi}, \tau_{yi}, \tau_{zi}, \tau_{ij} v_j + q\right]^T \\ \boldsymbol{Q} &= \left[p, v_x, v_y, v_z, T\right]^T \end{aligned} \tag{2}$$

Here ρ, v, and p are the density, velocity, and pressure of the fluid, respectively. τ is the viscous stress tensor, and q is the heat flux. In the above equations, H is total enthalpy per unit mass and is related to the total energy E by $H = E + p/\rho$, where E includes both internal and kinetic energies. The preconditioning matrix Γ is included in Eq. (1) to provide an efficient solution of the present axisymmetric compressible flow. This matrix is given by

$$\Gamma = \begin{bmatrix} \theta & 0 & 0 & 0 & \rho_T \\ \theta v_x & \rho & 0 & 0 & \rho_T v_x \\ \theta v_y & 0 & \rho & 0 & \rho_T v_y \\ \theta v_z & 0 & 0 & \rho & \rho_T v_x \\ \theta H - \delta & \rho v_x & \rho v_y & \rho v_z & \rho_T H + \rho C_p \end{bmatrix} \tag{3}$$

where ρ_T is the derivative of density with respect to temperature at constant pressure and δ =1 for an ideal gas and δ = 0 for an incompressible fluid. The parameter θ is defined as

$$\theta = 1/U_r^2 - \left(\rho_T H + \rho C_p\right) \tag{4}$$

In Eq. (4), the reference velocity U_r is chosen such that the eigenvalues of the system remain well conditioned with respect to the convective and diffusive timescales and C_p is the specific heat at constant pressure.

To close the governing equations, the k-ω SST which is a two-equation eddy-viscosity (Shear-Stress Transport) turbulent model [17-19] was employed in computations. This turbulence model is an effective blend of the robust and accurate formulation of the Wilcox's k-ω model in the near-wall region with the free-stream independence of the k-ε model in the far field.

The preconditioned governing equations were discretized spatially using a cell-centered finite volume scheme, in which the physical domain is subdivided into numerical cells and the integral equations are applied to each cell. A fully implicit method was implemented on the present multi-block spatial domain. The convective fluxes were formulated using the Roe's flux difference splitting scheme [20], and the third-order accuracy of this scheme was conceived from the original MUSCL (Monotone Upstream-Centered Schemes for Conservation Laws) [21] by blending a central differencing scheme and second-order upwind scheme. For the time derivatives in the governing equations, an implicit multistage time stepping scheme [22, 23], which is advanced from time t to time $t + \Delta t$ with a 2nd order Euler backward scheme for physical time and implicit pseudo-time marching scheme for inner iteration, was used.

The computational domain was meshed with quadrilateral elements with map scheme to yield a regular, structured grid. The grid nodes were placed in such a way that there were enough nodes near the throat and wall regions in order to capture the higher variable gradient accurately. The computational domain and boundary conditions for simulating the hysteresis phenomena in the divergent part of supersonic nozzle in the present study are illustrated in Fig. 3. The boundary conditions used are the inlet total pressure and the outlet static pressure respectively. The adiabatic no-slip conditions are applied to the solid walls. Here, in order to ensure the computational domain independent solutions, the domain downstream of the nozzle exit extends to the distance of $100D_t$ both in the x- and y-directions, respectively. A structured grid system was employed in the computations. The fineness of the computational grids was examined to ensure that the obtained solutions were independent of the grid employed. The solution was declared as converged when the residual for each variable becomes less than the chosen convergence criterion. In present simulations, the convergence criteria for the conserved variables are set to a value of 10^{-4}. Another convergence criterion is to check the conserved quantities directly through the computational boundaries. The net mass flux was investigated when there was an applicable imbalance through the computational boundaries.

3.2 Computational procedure

The hysteresis cycle has been often reported in several flow fields due to the nonlinear inherent in the flow under consideration. Such flows can be computed by numerical simulations [24], in which the flow boundary conditions are systematically changed to obtain each of the quasi-steady solutions. Figure 4 shows the computational procedure for the process of the startup transient, in which the pressure ratio is increased. The steady supersonic Laval nozzle flow of ϕ=ϕ_{st} is computed and the resulting solutions are used as the initial conditions for the first step of the process of the startup transient. In the second step, the pressure ratio is increased by $\Delta\phi$ and the computation is repeated until the transient process is completed, thus leading to a quasi-steady state, as indicated by the black circle. The computed quasi-steady solutions are used again as the initial conditions for the next step. Consequently the final quasi-steady solutions are obtained for the pressure ratio of ϕ_{fi}. On the contrary, for the process of the shutdown transients, in which the pressure ratio is decreased, the final quasi-steady solutions were used again as the initial conditions. In the present study, for shock wave in the straight part of the nozzle ϕ_{st}=2.17 and ϕ_{fi}=3.02, and for shock wave in the divergent part ϕ_{st}=1.42 and ϕ_{fi}=2.02, respectively. For

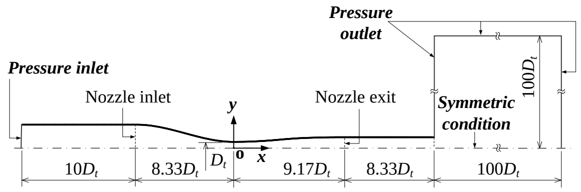


Fig 3. Schematic diagram of computational domain and boundary conditions

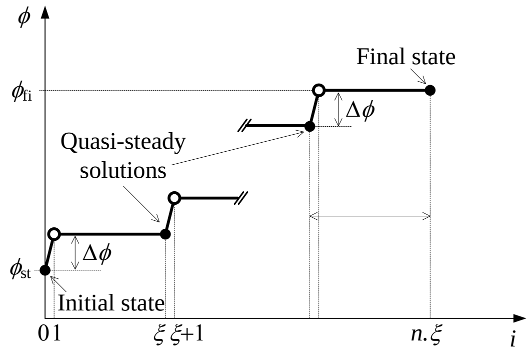


Fig 4. Computational procedure to simulate the processes of startup and shutdown transients

both cases $\Delta\phi$=0.029 and ξ=2000. Through such a series of computations, the quasi-steady solutions obtained during the startup and shutdown transients are compared to investigate the hysteretic behaviors of shock waves generated in the divergent and straight part of the supersonic nozzle.

4. RESULTS AND DISCUSSIONS

4.1 Experimental results

Figure 5 shows schlieren pictures of flow field in the divergent part of the supersonic nozzle. Value of rate of change of the pressure ratio with time $\Delta\dot{\phi}$ is 0.138 (1/s). In the figure, the left side sequence represents the increasing process of pressure ratio ϕ and the right side for the decreasing process of pressure ratio ϕ. As seen from these figures, there are differences between the locations of the first shock wave even in the same pressure ratio.

In the present study, investigating the hysteretic phenomena of shock wave in the divergent part of the nozzle the location L of the first shock wave was measured from the nozzle throat and the relationships between L/D_t and pressure ratio ϕ is depicted in Fig. 6. In this figure, symbols from (a) to (i) correspond to those in Fig. 5. As is evident from these figures, the existence of two values of L/D_t is confirmed in the ranges of ϕ =1.42 - 2.02.

In our previous researches on supersonic jet, the maximum mass flow rate in the jet within a certain pressure ratio changes with pressure ratio and this condition agrees with the range of occurrence of the hysteretic phenomenon. From this result, it is considered that the mass flow rate is related to the generation mechanism of hysteresis loop. However, as the flow has a nonlinear nature, the flow field is changed by influence of initial state of the entire flow field. As a result, in order

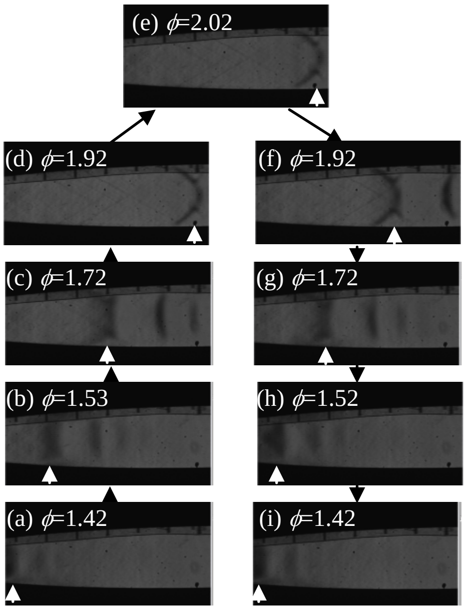


Fig 5. Schlieren photographs (Divergent part, $\Delta\dot{\phi}$ = 0.138 (1/s))

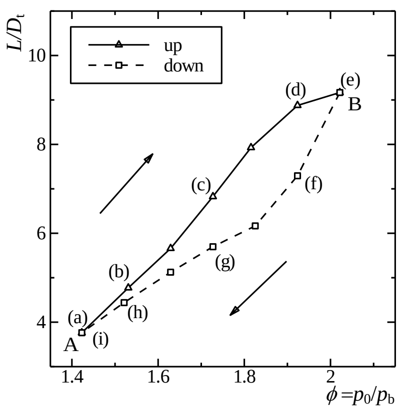


Fig 6. Hysteresis diagram for location of Mach disk

to understand the phenomenon, the flow inside and outside the nozzle should be considered.

4.2 Computational results

In order to investigate the hysteretic phenomena for shock wave in the divergent and straight part of the supersonic nozzle, a computational study is conducted in the range of pressure ratio from ϕ =1.35 to 2.17 and ϕ =2.17 to 3.02, respectively. Figures 7 and 8 show the computed density contours in the divergent and straight part of the nozzle for the range of ϕ =1.35 to 2.17 and ϕ =2.17 to 3.02, respectively. Here, the computed results have confirmed the formation of shock wave in the straight and divergent section of the nozzle, and some normal shocks are also formed in the flow field due to the sudden rise in temperature across the shock as explained in the previous research [15, 16]. During the startup transient of supersonic nozzle flow in Fig. 7, at pressure ratio ϕ =1.35 the oblique shock wave (first shock) is located just at the downstream of the nozzle throat. As ϕ increases to 2.17, the first shock wave moves downstream with stronger magnitude. At ϕ =2.17 which

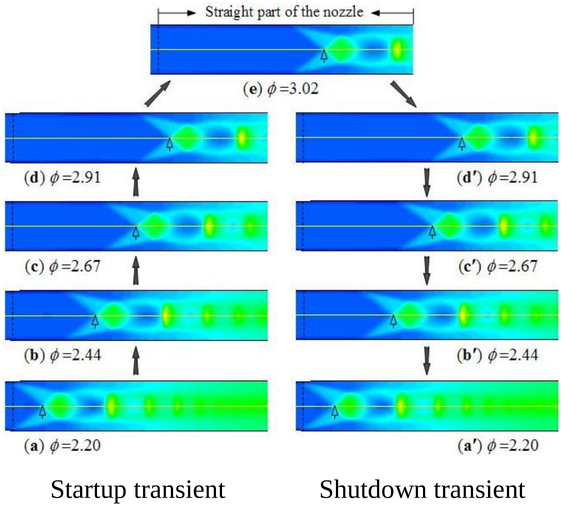


Fig 8. Density contours showing the hysteresis phenomena in the straight part of the nozzle

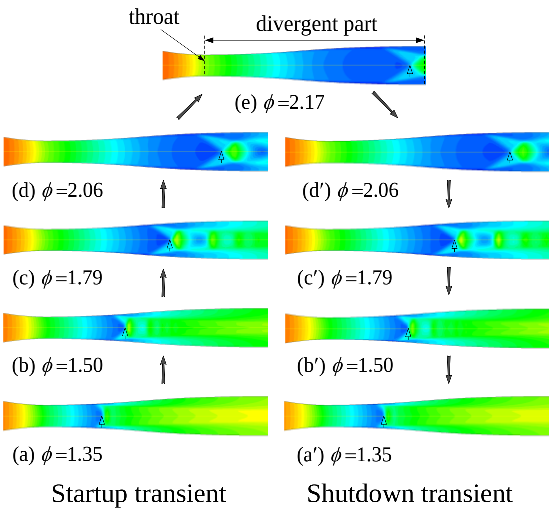
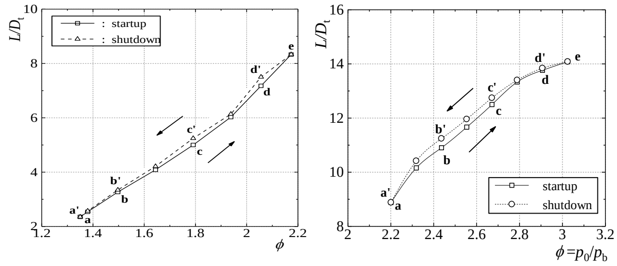


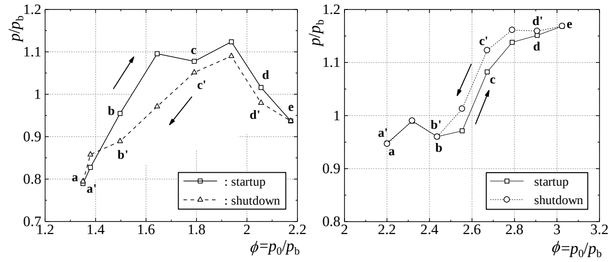
Fig 7. Density contours showing the hysteresis phenomena in the divergent part of the nozzle

corresponds to the final steady state in the startup transient, the computed flow field is nearly the same to that employed as the initial conditions in the shutdown transient. In the shutdown transient, as ϕ decreases again to 1.35, the shock wave moves upstream and the strength seems to be weaker than those found in the startup transient. From a series of computations, it is found that the location and strength of the first shock wave is significantly different in both the processes of the startup and shutdown transients. This clearly revealed that there exit a hysteretic behavior in the formation of shock wave in the divergent part of the supersonic nozzle for the range of pressure ratio ϕ =1.35 to 2.17. In the straight part of the nozzle, similar phenomena in the formation of shock waves have found for the range of ϕ =2.17 to 3.02 with comparatively stronger shock strength during the processes of shutdown transient.



(a) divergent part (b) straight part

Fig 9. Hysteretic behavior in the location of shock wave during startup and shutdown transients



(a) divergent part (b) straight part

Fig 10. Hysteretic behavior in shock strength during startup and shutdown transients

More quantitative data for hysteretic phenomena is presented in Figs. 9(a) and (b), where the location of first shock wave L/D_t, measured from the nozzle throat, in the divergent and straight part of the nozzle during the processes in the shutdown and startup transients are plotted against pressure ratio ϕ . In both cases, during the processes in the startup transient, the position of the first shock moves downstream with an increase in ϕ . On the contrary, in the processes of the shutdown transient, the shock position moves upstream with a decrease in ϕ . Hysteresis loop in the location of shock wave is found in both the transient processes.

Figures 10(a) and (b) show the hysteretic behavior in the shock strength which is corresponds to the static pressures p/p_b just behind the shock in the divergent and straight part of the supersonic nozzle, respectively. During the process of the startup transient for the straight part of the nozzle as shown in Fig. 10(b), the strength of the first shock increase with an increase in ϕ . On the contrary, in the process of the shutdown transient, the shock strength decreases with a decrease in ϕ . Therefore, the hysteretic behavior is produced during both the processes of the startup and shutdown transients. However, in the divergent part of the nozzle shown in Fig. 10(a), the shock strength increases with the increase in ϕ for the range pressure ratio ϕ =1.35 to 1.92 and after that it decreases again. Moreover, the shock strength in the divergent part of the nozzle is higher during the process of startup transient in comparison with the shutdown transient.

5. CONCLUSIONS

The present study deals with the experimental and computational works to investigate the hysteretic behavior in the formation of shock waves in supersonic nozzle. The axisymmetric, unsteady, compressible Navier-Stokes equations have been solved numerically to simulate the flow field concerning with the hysteretic behavior in both the processes of the shutdown and startup transients of supersonic nozzle flow. Hysteretic phenomena for the location of shock wave in a supersonic nozzle were investigated experimentally. As the results, hysteretic phenomenon for shock wave in the Laval nozzle was confirmed at a certain specific condition. The existence of hysteretic behavior in the formation, both the location and strength, of shock wave in the supersonic nozzle with a range of pressure ratio has also been confirmed numerically.

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7. NOMENCLATURE

Symbol	Meaning	Unit
D_e	nozzle exit diameter [mm]	(mm)
D_t	diameter at nozzle throat, characteristics length [mm]	(mm)
L	location of Mach disk [mm]	(mm)
M_e	Design Mach number	(-)
p	Static pressure	(Pa)
P_0	Stagnation pressure at plenum chamber	(Pa)
p_b	Atmospheric pressure	(Pa)
ϕ	Pressure ratio	(-)